Mathematics

Intermediate

Unit 2

Indices

Key Words:

Indices are just another word for "power"

5^4 — The base: 5; The index or power: 4

Two things you must remember about indices:

- Indices only apply to the number or letter they are to the right of – the base
  e.g. in abc^2, the squared only applies to the c, and nothing else. If you wanted the squared to apply to each term, it would need to be written as $(abc)^2$.
- Indices do not mean multiply
  e.g. 6^3 does not mean 6 x 3, it means 6 x 6 x 6

Multiplying Indices

Using index notation: $a^m \times a^n = a^{m+n}$

What it means: Whenever you are multiplying two terms with the same base, you can just add the powers!

Numbers: If there are numbers IN FRONT of your bases, then you must multiply those numbers together as normal

Examples

$x^3 \times x^4 = x^7$ ✓ Classic wrong answer: x^{12} ✗

$2^5 \times 2^3 = 2^8$ ✓ Classic wrong answer: 4^8 ✗

$3p^4 \times 2p^5 = 6p^9$ ✓ Classic wrong answer: $6p^{20}$ ✗

$2ab^2c \times 5ab^2c^3 = 10a^2b^4c^4$ ✓

Remember: if a base does not appear to have a power, the power is a 1.

e.g. $2ab^2c = 2a^1b^2c^1$

Dividing Indices

Using index notation: $a^m \div a^n = a^{m-n}$ Or $\frac{a^m}{a^n} = a^{m-n}$

What it means: Whenever you are dividing two terms with the same base, you can just subtract the powers!

Numbers: If there are numbers IN FRONT of your bases, then you must divide those numbers as normal

Examples

$x^{12} \div x^4 = x^8$ ✓ Classic wrong answer: x^3 ✗

$\frac{5^7}{5^3} = 5^4$ ✓ Classic wrong answer: 1^4 ✗

$\frac{20k^{10}}{5k^5} = 4k^5$ ✓ Classic wrong answer: $4k^2$ ✗

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Highest Common Factor, Lowest Common Multiple and Perfect Squares

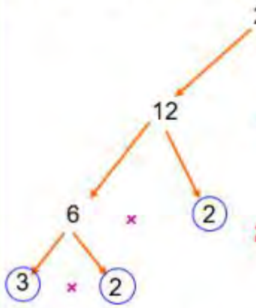
The Highest Common Factor (HCF) of two numbers, is the highest number that divides exactly into both numbers.

The Lowest Common Multiple (LCM) of two numbers, is the lowest number that is in the times table of both numbers.

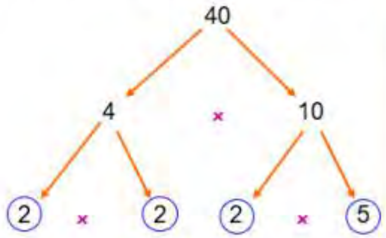
Finding the Highest Common Factor and Lowest Common Multiple Using Prime Factors.

Example: Find the LCM and HCF of 24 and 40

First, use Factor Trees to express your numbers as products of their prime factors:



24 = 2 x 2 x 2 x 3



40 = 2 x 2 x 2 x 5

Write out your answers:

24 = 2 x 2 x 2 x 3
40 = 2 x 2 x 2 x 5

Then in index form, like this

$24 = 2^3 \times 3$
$40 = 2^3 \times 5$

To get the Highest Common Factor, multiply together the numbers that appear in **BOTH** lists.

HCF = $2^3 = \mathbf{8}$

To get the Lowest Common Multiple, multiply together all the numbers that appear in either list, taking the highest power seen for each one. **Do not** include any duplicates.

LCM = $2^3 \times 3 \times 5 = \mathbf{120}$

Perfect Squares/Square Numbers

Example 1: $60 = 2^2 \times 3 \times 5$, is 60 a perfect square? If not, what do you need to multiply 60 by to make it a perfect square?

60 is not a perfect square as the indices on the prime factorisation are not all even numbers

$2^2 \times 3 \times 5$

2 is even

Remember, if you can't see an index number it means it is a 1, 1 is not an even number

To make 60 a perfect square we need to multiply it by:

$3 \times 5 = 15$

(This would then make all the indices even $2^2 \times 3^2 \times 5^2$)

Example 2: $400 = 2^4 \times 5^2$, is 400 a perfect square?

400 is a perfect square as the indices on the prime factorisation are all even numbers

4 is even 2 is even

$2^4 \times 5^2$

Example 3: The number 32,768 is equal to 2^{15}. Explain how this tells you that 32,768 is not a square number.

The index number, 15, is not an even number, so 32,768 is not a square number.

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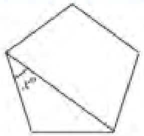
Unit 3

Regular Polygon Questions

Example 1:

The diagram shows a regular pentagon.
Work out the value of x.



Using the rule: Sum of interior angles = $(n - 2) \times 180°$

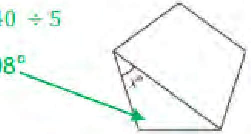
= $(5 - 2) \times 180°$

= 540°

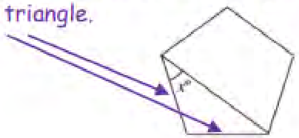
Using the rule: One interior angle = Sum of interior angles $\div n$

= $540 \div 5$

= 108°



As the shape is a regular shape, the 2 sides of the triangle must be equal, it is an isosceles triangle.

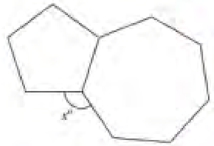


So, $x = \frac{180 - 108}{2}$

$x = 36°$

Example 2:

The diagram shows a regular pentagon and a regular heptagon.
Work out the value of x.



Angles around a point add to 360°, so x + one interior angle of the pentagon + one interior angle of the heptagon = 360°

Pentagon:

Using the rule: Sum of interior angles = $(n - 2) \times 180°$

= $(5 - 2) \times 180°$

= 540°

Using the rule: One interior angle = Sum of interior angles $\div n$

= $540 \div 5$

= 108°

Heptagon:

Using the rule: Sum of interior angles = $(n - 2) \times 180°$

= $(7 - 2) \times 180°$

= 900°

Using the rule: One interior angle = Sum of interior angles $\div n$

= $900 \div 7$

= 128.6° (1 d.p.)

$x = 360 - 108 - 128.6$

$x = 123.4°$

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Unit 3

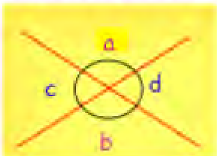
Parallel Lines

Parallel lines are lines which never meet, and always keep a perfectly equal distance apart.

Remember: Only assume lines are parallel if they have those little arrows on them:

Opposite Angles

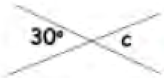
Fact: Opposite Angles are equal



a = b

c = d

Example: Calculate angle c.



c = 30°

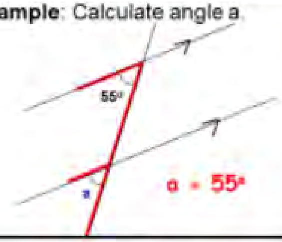
How to spot it: Find two continuous straight lines crossing at a point. The pairs of angles opposite each other will be equal

Note: Using Fact 2, all the angles around that point will add up to 360°

Corresponding Angles

Fact: Corresponding Angles are equal

Example: Calculate angle a.



a = 55°

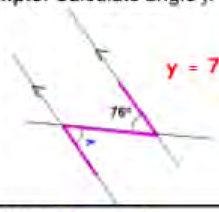
How to spot it: Look for the F shape, the angles underneath the arms of the F are equal

Note: The arms of the F must definitely be Parallel lines!

Alternate Angles

Fact: Alternate Angles are equal

Example: Calculate angle y.



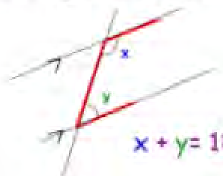
y = 76°

How to spot it: Look for the Z shape, the angles "inside" the Z are equal

Note: The top and bottom of the Z must be Parallel Lines!

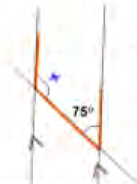
Interior Angles

Fact: Interior Angles add up to 180°



x + y = 180°

Example: Calculate angle x.



x = 180 - 75
= 105°

How to spot it: Look for the C shape, the angles underneath the top and bottom of the C add up to 180°

Note: The top and bottom of the C must definitely be Parallel lines!

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Unit 9

Simplifying in Algebra

Key Words:

Term: This is any part of an expression or equation that involves a letter.

e.g. $4m$, $-2r$, and p are all terms

Expression: This is a collection of terms, sometimes including numbers as well, it does not have an equals sign.

e.g. $4m + 2r$ and $8z - 5p + 6q^2 - 7$ are all expressions

Equation: This is like an expression but it contains an equals sign.

e.g. $4m + 2r = 7$ and $8z + 6q^2 - 7 = a$ are all equations

Directed numbers

+ + → +

− − → +

+ − → −

− + → −

Eg $a = -5$ and $b = 2$

$a^2 = a \times a = -5 \times -5 = 25$

$b + a = 2 + -5 = -3$

You can add or subtract LIKE TERMS but you cannot add or subtract DIFFERENT TERMS.

A LIKE TERM is a term that contains the exact same letter (or letters) as another term

e.g. $m + m = 2m$ $\quad 3p + 2p = 5p$ $\quad 16t^2 - 4t^2 = 12t^2$ $\quad 10pq - 7pq = 3pq$

3 lots of something, plus 2 lots of something, gives you 5 lots of something

16 lots of something, minus 4 lots of something, gives you 12 lots of something

BUT...

$m + p$ Does Not = mp $\quad 3r + 2t$ Does Not = $5rt$

Because the terms are different!

Mathematics

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Unit 9

Simplifying Expressions

Adding and Subtracting

Note: To simplify an expression, draw boxes around all the LIKE TERMS and deal with each set of like terms on their own.

Example 1: Simplify $4m + 2p - m + 6p$

Remember: Draw each box around the term and the sign in front of the term.

$\boxed{4m}\ \boxed{+\ 2p}\ \boxed{-\ m}\ \boxed{+\ 6p}$

We have:

$4m - m = 3m$

$2p + 6p = 8p$

So: $4m + 2p - m + 6p = 3m + 8p$

Note: if you cannot see a sign in front of a term, then just assume it is a PLUS

Example 2: Simplify $4t^2 - 5t - 2t - 3t^2$

Remember: t and t^2 are DIFFERENT!

$\boxed{4t^2}\ \boxed{-\ 5t}\ \boxed{-\ 2t}\ \boxed{-\ 3t^2}$

We have:

$4t^2 - 3t^2 = t^2$

$-5t - 2t = -7t$

Note: write this instead of $1\,t^2$

So: $4t^2 - 5t - 2t - 3t^2 = t^2 - 7t$

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Unit 10

Substitution in Algebra

Substitution is where you are told the value of a letter and you substitute this into an expression or equation.

e.g. Find the value of $5x$ when $x = 7$. $5x$ means 5 x x = 5 x 7 = 35.

- Always apply BIDMAS/BODMAS
- Use brackets for powers
- For fractions, work out the top and bottom separately.

Example 1: Evaluate (find the **value** of) the expressions, given that:

$a = 2,\ b = 3,\ c = -5, d = -1$

a) $5a = 5 \times 2$
$= \mathbf{10}$

b) $3b - 2c = 3 \times 3 - 2 \times (-5)$
$= 9 + 10$
$= \mathbf{19}$

c) $4b^2 + d = 4 \times 3^2 + (-1)$
$= 4 \times 9 - 1$
$= 36 - 1$
$= \mathbf{35}$

d) $3a^3 = 3 \times (2)^3$
$= 3 \times 8$
$= \mathbf{24}$

e) $\frac{5cd}{a+b} = \frac{5 \times (-5) \times (-1)}{2+3}$
$= \frac{25}{5}$
$= \mathbf{5}$

f) $c^2 + abd = (-5)^2 + 2 \times 3 \times (-1)$
$= 25 - 6$
$= \mathbf{19}$

Example 2: Evaluate (find the **value** of) the expressions, given that: (*calculator questions*)

$a = 1.2,\ b = \frac{1}{9},\ c = -3.65$

a) $4b - 6c + a^2 = 4 \times \frac{1}{9} - 6 \times (-3.65) + (1.2)^2$
$= \frac{4}{9} + 21.9 + 1.44$
$= \mathbf{23.78\dot{4}}$

b) $\sqrt{\frac{a + 4c}{3b + c}} = \sqrt{\frac{1.2 + 4 \times (-3.65)}{3 \times \frac{1}{9} + (-3.65)}}$

$= \sqrt{\frac{-13.4}{-3.31\dot{6}}}$

$= \sqrt{4.0402010051}$

$= 2.0100251255$

Learn how to do these in one step using your *scientific* calculator.

Example 3: Use the formula $P = 5A - 6B$ to find the value of:

a) P when $A = 7$ and $B = -4$.

$P = 5A - 6B$

$P = 5 \times 7 - 6 \times (-4)$

$P = 35 + 24$

$P = 59$

b) A when $B = 3$ and $P = 37$

$P = 5A - 6B$

$37 = 5A - 6 \times 3$

$37 = 5A - 18$

$37 + 18 = 5A$

$55 = 5A$

$\frac{55}{5} = A \qquad A = 11$

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Unit 11

Example 4: $24 - 3m = 6$

$$24 - 3m = 6$$

Move the $+24$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$-3m = 6 - 24$$

$$-3m = -18$$

Remember, even though it is a - 3, it is being multiplied by the m, so the opposite / inverse operation is a divide

Move the $\times(-3)$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$m = \frac{-18}{-3}$$

$$m = 6$$

Example 5: $7y + 3 = 10y - 6$

$$7y + 3 = 10y - 6$$

Move the $+3$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$7y = 10y - 6 - 3$$

$$7y = 10y - 9$$

Move the $+10y$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$7y - 10y = -9$$

$$-3y = -9$$

Move the $\times(-3)$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$y = \frac{-9}{-3}$$

$$y = 3$$

Example 6: $5(x - 3) = 4(x + 2)$

$$5(x - 3) = 4(x + 2)$$

Expand the brackets on both sides

$$5x - 15 = 4x + 8$$

Move the -15 over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$5x = 4x + 8 + 15$$

$$5x = 4x + 23$$

Move the $+4x$ over the equals sign to the opposite side and change the sign to the opposite / inverse operation

$$5x - 4x = 23$$

$$x = 23$$

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Unit 11

Method 2: Balancing equations

Golden Rule: Whatever you do to one side of the equation, you must do exactly the same to the other side to keep the equation in balance

Step 1: If they are not already, get all your unknown letters on one side of the equation (NOT on the bottom of fractions and avoiding negatives).

Step 2: Begin undoing the operations that were done to your unknown letter, by thinking about the order that things were done to the letter

Step 3: Use inverse operations to do this until you are left with just your unknown letter on one side, and the answer on the other

Step 4: Check your answer using substitution to make sure your answer is right.

Example 1: $7p - 3 = 32$

Step 1: The unknown letter (p) only appears on the left-hand side of the equation, there is no negative sign in front of it, and it is not on the bottom of a fraction.

$$7p - 3 = 32$$

Step 2: What order were things done to p? First it was multiplied by the 7, then 3 was subtracted.

Step 3: To undo the operations, we start with the last one, working our way backward and apply the inverse (opposite) operation to both sides:

$$7p - 3 = 32$$
$$+3 \qquad +3$$

The last operation was -3, so the opposite / inverse operation is +3, remembering the rule whatever you do to one side of the equation you do to the other.

$$7p = 35$$

Now divide both sides by 7

$$\div 7 \qquad \div 7$$

$$p = 5$$

Step 4: Check if the answer is right. Substitute $p = 5$ into the initial equation.

When p = 5

$$7p - 3 = 7 \times 5 - 3 = 35 - 3 = 32$$

Example 2: $24 - 3m = 6$

Step 1: The unknown letter (m) only appears on the left-hand side of the equation, it's not on the bottom of a fraction, but it does have a negative sign in front of it.

We can use inverse operations to cancel out the -3m, we just need to add 3m to both sides.

$$24 - 3m = 6$$
$$+3m \qquad +3m$$
$$24 = 6 + 3m$$

Step 2: What order were things done to m? First it was multiplied by the 3, then 6 was added.

Step 3: To undo the operations, we start with the last one, working our way backward and apply the inverse (opposite) operation to both sides:

$$-6 \qquad -6$$

The last operation was +6, so the opposite / inverse operation is -6, remembering the rule whatever you do to one side of the equation you do to the other.

$$18 = 3m$$

Now divide both sides by 3

$$\div 3 \qquad \div 3$$

$$6 = m \quad \text{or} \quad m = 6$$

Step 4: Check if the answer is right. Substitute $m = 6$ into the initial equation.

When m = 6

$$24 - 3m = 24 - 3 \times 6 = 24 - 18 = 6$$

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Unit 11

Example 3: $6+\frac{k}{5} = -1$

$$6+\frac{k}{5} = -1$$

$-6 \qquad\qquad -6$

$$\frac{k}{5} = -7$$

$\times 5 \qquad\qquad \times 5$

$$k = -35$$

Check: Substitute k= −35 into the original equation.

$$6+\frac{k}{5} = 6+\frac{-35}{5} = 6+-7 = 6-7 = -1$$

Example 4: $2(3r+6) = 36$

$$2(3r+6) = 36$$

Expand brackets

$$6r + 12 = 36$$

$-12 \qquad\qquad -12$

$$6r = 24$$

$\div 6 \qquad\qquad \div 6$

$$r = 4$$

Check: Substitute $r = 4$ into the original equation.

$$2(3r + 6) = 2(3 \times 4 + 6) = 2(12 + 6) = 2 \times 18 = 36$$

Example 5: $7y + 3 = 10y - 6$

$$7y + 3 = 10y - 6$$

$-7y \qquad\qquad -7y$

$$3 = 3y - 6$$

$+6 \qquad\qquad +6$

$$9 = 3y$$

$\div 3 \qquad\qquad \div 3$

$3 = y$ or $y = 3$

Check: $10y - 6 = 10 \times 3 - 6 = 24$

Example 6: $5(x - 3) = 4(x + 2)$

$$5(x - 3) = 4(x + 2)$$

expand $\qquad\qquad$ expand

5⍰ – 15 = 4⍰ + 8

$-4x \qquad\qquad -4x$

$$x - 15 = 8$$

$+15 \qquad\qquad +15$

$$x = 23$$

Check: Substitute $c = 23$ into the original equation.

$$5(23 - 3) = 4(23 + 2)$$

$$5 \times 20 = 4 \times 25$$

$$100 = 100$$

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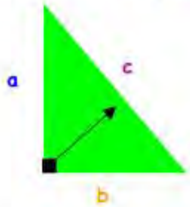
Unit 13

Finding the Length of the Hypotenuse (the Longest Side)

1. Label the hypotenuse **c**, and the other sides **a** and **b**

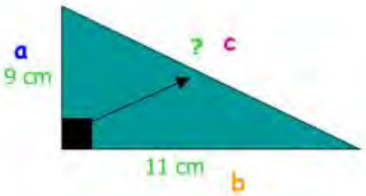
2. Use the following formulae:

$$c^2 = a^2 + b^2$$



3. Replace the letters with the numbers you have been given, and solve.

Example:



The side we are looking for is the hypotenuse.

Step 1. Label the sides

Step 2. Use the formula: $c^2 = a^2 + b^2$

Step 3. Put in the numbers:

$$c^2 = 9^2 + 11^2$$

$$c^2 = 81 + 121$$

$$c^2 = 202$$

$$c = \sqrt{202}$$

$$c = 14.2cm \quad \text{(1dp)}$$

Square root both sides!

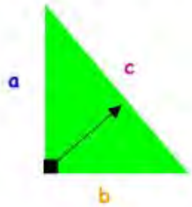
Check: The hypotenuse is the longest side of the triangle, 14.2cm is longer than the other two sides.

Finding the Length of a Shorter Side

1. Label the hypotenuse **c**, label the side you want to find **a**, and the other side **b**

2. Use the following formulae:

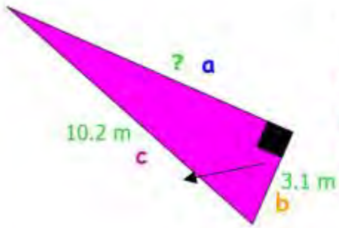
$$a^2 = c^2 - b^2$$



3. Replace the letters with the numbers you have been given, and solve.

Note: As mentioned before, this is just a different arrangement of: $c^2 = a^2 + b^2$

Example:



The side we are looking for is one of the shorter sides

Step 1. Label the sides

Step 2. Use the formula: $a^2 = c^2 - b^2$

Step 3. Put in the numbers:

$$a^2 = 10.2^2 - 3.1^2$$

$$a^2 = 104.04^2 - 9.61^2$$

$$a^2 = 94.43$$

$$a = \sqrt{94.43}$$

$$a = 9.72m \quad \text{(2dp)}$$

Square root both sides!

Check: a = 9.72m which is shorter than the hypotenuse (the longest side).

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Unit 17

Perimeter, Area, Volume and Density

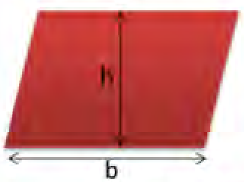
The area of a 2D shape is the space inside it. It is measured in units squared e.g. cm^2

The **perimeter** of a shape is the distance around the edge of the shape. Units of length are used to measure perimeter e.g. mm, cm, m

A **compound shape** is a shape made from other shapes joined together.

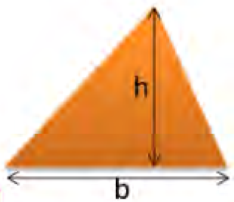
Formulas for Area:

Parallelogram



A = b × h

Triangle



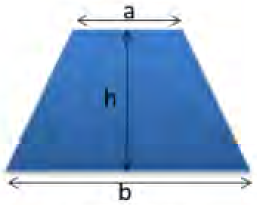
$A = \frac{b \times h}{2}$

Note: You must use the perpendicular height

Rectangle / Square

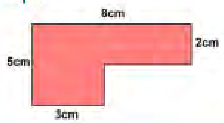
A = l × w

Trapezium



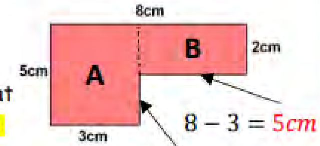
$A = \frac{(a+b) \times h}{2}$

Example 1: Find the perimeter and area of the compound shape



Area

Step 1: Split the shape into shapes that you can find the area of



$8 - 3 = 5cm$

$5 - 2 = 3cm$

Step 2: Find the missing lengths of sides

Step 3: Work out the area of each shape

Step 4: Work out the total area, remembering the units

Area A $= (5 \times 3)$
$= 15cm^2$

Area B $= (2 \times 5)$
$= 10cm^2$

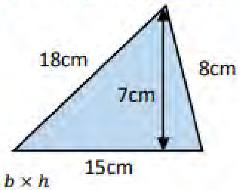
Total area $= 15 + 10$
$= 25cm^2$

Perimeter

Add up all the outside edges

Perimeter $= 3 + 5 + 8 + 2 + 5 + 3$
$= 26cm$

Example 2: Find the area of the triangle

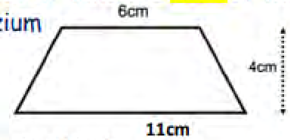


Area $= \frac{b \times h}{2}$

$= \frac{15 \times 7}{2}$

$= 52.5cm^2$

Example 3: Find the area of the trapezium



Area $= \frac{(a+b) \times h}{2}$

$= \frac{(6+5) \times 4}{2}$

$= 22cm^2$

Mathematics

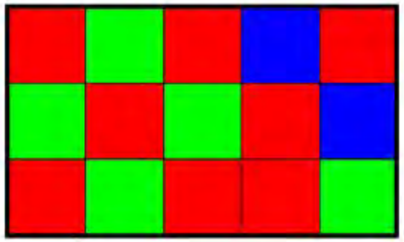
Intermediate

Unit 18

Ratio and Proportion

Writing Ratios

Ratios require the use of a colon **:**



The ratio of red squares to green squares is:

8 : 5

Because for every 8 red squares, there are 5 green:

The ratio of green squares to red squares is:

5 : 8

The ratio of blue squares to red squares is:

2 : 8

Simplifying Ratios

Method Just like with fractions, whatever you multiply/divide one side by, make sure you do the exact same to the other side. Keep dividing until each side has no common factors

Example 1: Simplify 14 : 21

We are looking for factors common to both sides, let's try 7.

Divide both sides by 7

$$14 : 21 \xrightarrow{\div 7} 2 : 3$$

Check: Are there are any other common factors to simplify it further? No, we have simplified it as far as possible.

Example 2: Simplify 60 : 45

We are looking for factors common to both sides, let's try 5.

Divide both sides by 5

$$60 : 45 \xrightarrow{\div 5} 12 : 9$$

Check: Are there are any other common factors to simplify it further? Yes, 3 is a common factor to both sides.

Divide both sides by 3

$$12 : 9 \xrightarrow{\div 3} 4 : 3$$

Check: Are there are any other common factors to simplify it further? No, we have simplified it as far as possible.

Note: For example 2 we could have divided both sides by 15 to start, which would have given us our answer of 4 : 3 in one step. It does not matter which way you choose, just make sure you simplify as much as possible.

Mathematics

Intermediate

Unit 19

Ordering Fractions, Decimals and Percentages

To order a mix of fractions, decimals, and percentages you need to first convert all the numbers to the same form, either fractions, decimals, or percentages.

Note: Ascending Order means smallest to largest.

Descending Order means largest to smallest.

Example:

Put the following in ascending order

56% $\frac{3}{4}$ 0.871 23% $\frac{6}{7}$

To order these, convert them all to decimals.

56%	$\frac{3}{4}$	0.871	23%	$\frac{6}{7}$
0.56	0.75	0.871	0.23	0.857...
2	3	5	1	4

Then write the correct order but as they were in the original question.

23% 56% $\frac{3}{4}$ $\frac{6}{7}$ 0.871

Here are some equivalent fractions, decimals, and percentages you should know.

F	D	P
$\frac{1}{100}$	0.01	1%
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.2	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{3}$	$0.\dot{3}$	$33.\dot{3}\%$
$\frac{2}{3}$	$0.\dot{6}$	$66.\dot{6}\%$

Recurring Decimals

Some decimals **terminate**, which means the decimals do not recur, they just stop. For example, 0.75.

A **recurring decimal** exists when decimal numbers repeat forever.

Convert $\frac{8}{11}$ into a decimal using your calculator. A calculator displays this as $0.\dot{7}\dot{2}$ or 0.7272727272......

The digits 2 and 7 repeat infinitely. This is an example of a **recurring decimal**.

We can show this by writing dots above the 7 and the 2 (the numbers that recur).

If you had to convert into a recurring decimal without the calculator, you would need to use the bus shelter method Write $\frac{5}{6}$ as a decimal $6\,\overline{)5.0000}$ = 0.8333 So, $\frac{5}{6} = 0.8\dot{3}$

Here are some more examples of recurring decimals:

$\frac{4}{9} = 0.\dot{4}$	This decimal is made up of an infinite number of repeating 4s.
$\frac{5}{6} = 0.8\dot{3}$	This decimal starts with an 8 and is followed by an infinite number of repeating 3s.
$\frac{2}{7} = 0.\dot{2}8571\dot{4}$	In this decimal, the six digits 285714 repeat an infinite number of times in the same order.
$\frac{9}{22} = 0.4\dot{0}\dot{9}$	This decimal starts with a 4. The two digits 09 then repeat an infinite number of times.

Mathematics

Intermediate

Unit 21

Solving Trigonometry Problems

When finding an angle, remember you need to use one of the inverse operations either
sin^{-1}, cos^{-1} or tan^{-1}

You will need to press SHIFT on your calculator first.

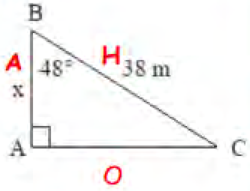
Example 1: Find the length of AB (this can be labelled x)

Step 1: Label the sides O, A and H

Step 2 and 3: Tick or highlight what you have been given and what you have been asked to work out

SOH CAH TOA

Step 4: Decide whether you need sin, cos or tan. Looking above the only one that contains both A and H is **cos**



$$\cos\theta = \frac{A}{H}$$

$$\cos 48 = \frac{x}{38}$$

$$38 \times \cos 48 = x$$

$$x = 25.4m\ (1dp)$$

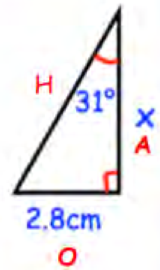
Example 2: Find the length of x

Step 1: Label the sides O, A and H

Step 2 and 3: Tick or highlight what you have been given and what you have been asked to work out

SOH CAH TOA

Step 4: Decide whether you need sin, cos, or tan. Looking above the only that contains both A and O is **tan**



$$\tan\theta = \frac{O}{A}$$

$$\tan 31 = \frac{2.8}{x}$$

Note: Because the unknown x is on the bottom you would need to multiply up by x and divide by $\tan 31$ but it is easier just to swap them.

$$x = \frac{2.8}{\tan 31}$$

$$x = 4.66cm\ (2dp)$$

(on the calculator you can press 2.8 ÷ tan31 =)

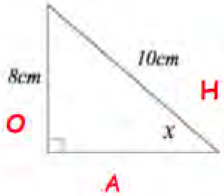
Example 3: Find the size of the angle x

Step 1: Label the sides O, A and H

Step 2: Tick or highlight what you have been given

SOH CAH TOA

Step 4: Decide whether you need sin, cos, or tan. Looking above the only that contains both O and H is **sin**



$$\sin x = \frac{O}{H}$$

$$\sin x = \frac{8}{10}$$

(on the calculator you can press shift sin, 8 ÷ 10 =)

$$x = \sin^{-1}\left(\frac{8}{10}\right)$$

$$x = 53.1°\ (1dp)$$

Do not forget to round your answers to an appropriate degree of accuracy.

Mathematics

Intermediate

Unit 26

Notice in example 4 the sequence is non-linear (the difference between terms are different). That is because it was a **Quadratic n^{th} term**, the n was squared.

Now we will look at finding the n^{th} term. There are two methods.

Generating a Sequence from the n^{th} Term

Example 4: The n^{th} term for a sequence is $2n^2 - 3$.

What are the first 5 terms of the sequence?

Position	1	2	3	4	5
Workings	$2\times(1)^2-3$	$2\times(2)^2-3$	$2\times(3)^2-3$	$2\times(4)^2-3$	$2\times(5)^2-3$
Sequence	-1	5	15	29	47

Apply BIDMAS. As the square is only attached to the n, square the n first then multiply by 2!

The sequence is -1, 5, 15, 29, 47...

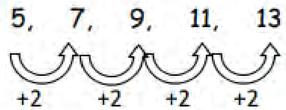
Finding the n^{th} term (linear sequence)

Example 5a: Find the n^{th} term of this sequence.

5, 7, 9, 11, 13

Option 1:

Note the difference between each term



This number goes in front of the n $2n$

Subtract your number from the first sequence number 5 - 2 = 3
This is the second part of your nth term

The n^{th} term is $2n + 3$

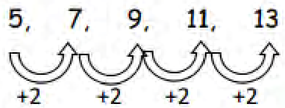
Finding the n^{th} term (linear sequence)

Example 5b: Find the n^{th} term of this sequence.

5, 7, 9, 11, 13

Option 2:

Note the difference between each term



This number goes in front of the n $2n$

Substitute in 1, work out how to get from your number to the first term in the sequence. 2 x 1 = 2 We need 5 so we add 3

The n^{th} term is 2n + 3

Mathematics

Intermediate

Unit 27

Tip: In the next section you may end up with answers that look like standard form but do not obey the rules. Therefore, you will need to convert to standard form.

e.g. 16×10^4 — 16 is not between 1 and 10!

Option one:

Convert to an ordinary number then convert to the accepted version of standard form

$160000 = 1.6 \times 10^5$

Option two:

16×10^4

Write as standard form → $1.6 \times 10^1 \times 10^4$

1.6×10^5

Remember your rules of indices

Standard form can be written on a scientific calculator with the 'exponent' button. The exponent button may be shown as EXP, EE or $\times 10^x$.

Multiplying and Dividing in standard form

Multiply or divide the front numbers, then add or subtract the indices using the rules of indices.

Examples:

$(3 \times 10^4) \times (2 \times 10^2)$
$3 \times 2 \times 10^4 \times 10^2$
6×10^6

$(8 \times 10^4) \div (2 \times 10^2)$
$8 \div 2 \quad 10^4 \div 10^2$
4×10^2

$(3 \times 10^4) \times (4 \times 10^2)$
$3 \times 4 \times 10^4 \times 10^2$
12×10^6
$1.2 \times 10^1 \times 10^6$
1.2×10^7

$\frac{(3 \times 10^{-10})}{(6 \times 10^5)}$
$3 \div 6 \quad 10^{-10} \div 10^5$
0.5×10^{-15}
$5 \times 10^{-1} \times 10^{-15}$
5×10^{-16}

Adding and subtracting in standard form

When adding or subtracting in standard form we can either convert to ordinary numbers first or convert the numbers to have the same power of 10.

Example 1 $(2.3\times10^4) + (4.31\times10^5)$

$(2.3\times10^4) \rightarrow 23000$
$(4.31\times10^5) \rightarrow 431000$

We must change both numbers into normal numbers.

Now we line our digits up carefully and add...

```
  431000
+  23000
  454000
```

Usually you will then be asked to convert your answer back into Standard Form...

$454000 = 4.54 \times 10^5$

Example 2 $(1.2 \times 10^{17}) - (6.4 \times 10^{16})$ $\quad 1.2 \times 10^{17} \rightarrow 12 \times 10^{16}$

By converting the first number, both now have the same power of 10 (10^{16}). The front numbers can now be subtracted.

$(12 \times 10^{16}) - (6.4 \times 10^{16}) = 5.6 \times 10^{16}$

Make sure your final answer is in standard form.

Mathematics

Intermediate

Unit 33

Simultaneous Equations

Simultaneous Equations are two equations, each containing two unknown letters, and you must use both equations, to find the value of your unknown letters.

Key Point: The values you find for your unknown letters must make BOTH equations balance - you can check your answers and make sure that you have got it right.

Method for Solving Simultaneous Equations:

Step 1: If you need to, re-arrange your equations so they are in the same form

Step 2: Write one equation underneath the other, lining up the unknown letters

Step 3: Choose one of the unknown letters and use your algebra skills to change one or both of the equations to make sure there are the same number (don't worry about sign) of your chosen letter in each equation. Your chosen letter becomes your Key Letter.

Step 4: Put a box around your Key Letters and their sign

Step 5: Follow this rule:

If the signs are the same, subtract the two equations

If the signs are different, then add the two equations

Step 6: If you have done this correctly, your Key Letter should cancel out and you should be left with just one equation with one unknown

Step 7: Solve this equation to work out the value of the unknown letter

Step 8: Choose one of the original equations and substitute in the answer you found in **Step 7.** to work out the value of the other letter.

Step 9: Check your answers are correct using the equation you **did not** choose in **Step 8.**

Example 1: Solve $3x + y = 19$ and $x + y = 9$

1. The equations are in the same form, some x's and some y's, equal a number.

2. Write the second equation underneath the first.

$$3x + y = 19 \quad (1)$$
$$x + y = 9 \quad (2)$$

3. There are already the same number of y's in both equations (there is an invisible 1 in front of both), so let us choose the y's to be our Key Letters.

4. Put a box around our Key Letters, and their signs.

(1) – (2)

$$\begin{array}{r} 3x \boxed{+y} = 19 \\ x \boxed{+y} = 9 \\ \hline 2x \quad = 10 \end{array} \; -$$

5. The signs of our Key Letters are the same (both +) so we must subtract equation (2) from equation (1)

6. Our Key Letters have cancelled out.

7. Solve the equation.

$$2x = 10$$
$$x = \frac{10}{2}$$
$$x = 5$$

8. Substitute this value in one of the original equations to find the value of the other unknown letter.

Substitute in (2) :

$$5 + y = 9$$
$$y = 9 - 5$$
$$y = 4$$

9. We now have our two solutions. Check them using the equation we did not choose in 8.

Check in (1) :

$$3 \times 5 + 4 = 9$$

The solutions are, $x = 5$ and $y = 4$

Mathematics

Intermediate

Unit 33

Example 2: $3x-2y=3$ and $2x+2y=12$

$$3x\boxed{-2y}=3 \quad (1)$$
$$2x\boxed{+2y}=12 \quad (2)$$

Signs of Key Letters are different so add

$$\begin{aligned} 3x-2y&=3\\ 2x+2y&=12 \quad +\\ \hline 5x\quad\ &=15 \end{aligned}$$

$$x=3$$

Substitute in (2):

$$2\times3+2y=12$$
$$6+2y=12$$
$$2y=12-6$$
$$2y=6$$
$$y=\tfrac{6}{3}$$
$$y=2$$

Check in (1):

$$3\times3-2\times3=3$$

Solutions are $x=3$ and $y=3$

Example 3: $2x+3y=7$ and $3x+5y=18$

$$2x+3y=7 \quad (1)$$
$$3x+5y=18 \quad (2)$$

Multiply (1) by 3: $\boxed{6x}+9y=21$

Multiply (2) by 2: $\boxed{6x}+10y=36$

Signs of Key Letters are the same so subtract

$$\begin{aligned} 6x+10y&=36 \quad -\\ 6x+9y&=21\\ \hline y&=15 \end{aligned}$$

Substitute in (1): $2x+3\times15=7$

$$2x+45=7$$
$$2x=7-45$$
$$2x=-38$$
$$x=-\tfrac{38}{2}$$
$$x=-19$$

Check in (2):

$$3\times(-19)+5\times15=18$$

Solutions are $x=-19$ and $y=15$

Example 4: $7x-2y=-20$ and $3x=6-4y$

Rearranging the second equation gives: $3x+4y=6$

$$7x-2y=-20 \quad (1)$$
$$3x+4y=6 \quad (2)$$

Multiply (1) by 2: $14x-4y=-40$

Signs of Key Letters are different so add:

$$\begin{aligned} 14x\boxed{-4y}&=-40\\ 3x\boxed{+4y}&=6 \quad +\\ \hline 17x\quad\ &=-34 \end{aligned}$$

$$x=-\tfrac{34}{17}$$
$$x=-2$$

Substitute in (2): $3\times(-2)+4y=6$

$$-6+4y=6$$
$$4y=6+6$$
$$4y=12$$
$$y=\tfrac{12}{4}$$
$$y=3$$

Check in (1):

$$7\times(-2)-2\times3=-20$$

Solutions are $x=-2$ and $y=3$

Mathematics

Intermediate

Unit 38

Trial and Improvement

Trial and improvement is a method for finding an *approximate* solution to an equation that you would not be able to solve using your normal way.

It will tell you in the question when to use trial and improvement.

The idea of trial and improvement is to keep trying different values of x to get you closer and closer to the solution. If you remember the method, then they are easy marks to pick up.

Method

1. Draw a table for your values (*see example*).
2. Substitute **two** values into the equation: These two values are usually given to you in the question and will result in an answer that is too big and one that is too small i.e. opposite cases.
3. Substitute the next value into the equation which is between the previous two values: Choose the middle value (or close to the middle) of what you've already substituted.
4. If the last value is too small substitute in the next higher consecutive value correct to 1.d.p into the equation. If the last value is too big substitute in the next lower consecutive value correct to 1.d.p into the equation.
5. Keep going until you have two values next to each other in which one is too big, and one is too small.
6. **Substitute in the value** halfway between these two numbers.
   If the halfway answer is too small choose the bigger value correct to 1.d.p
   If the halfway answer is too big choose the smaller value correct to 1.d.p
   E.g. for 2.3 and 2.4 you would try 2.35 to see if the correct answer was between 2.3 and 2.35 (so you'd give 2.3 to 1dp) or between 2.35 and 2.4 (hence you'd give 2.4 as your answer to 1dp).

Example

The solution to the equation $x^3 + 9x = 40$ lies between 2 and 3.

Use trial and improvement to find the solution correct to 1dp.

Values from the question

x	$x^3 + 9x$	Comment
2	$2^3 + 9(2) = 26$	Too small
3	$3^3 + 9(3) = 54$	Too big
2.5	$2.5^3 + 9(2.5) = 38.125$	Too small
2.6	$2.6^3 + 9(2.6) = 40.976$	Too big
2.55	$2.55^3 + 9(2.55) = 39.531375$	Too small

This means x is between 2.55 and 2.6

$\therefore x = 2.6$

Mathematics

Intermediate

Unit 42

Listing Outcomes

You might be asked to list all the possible outcomes for two or more events.

Example: List all the 3-digit numbers that can be made using the digits 3, 6, and 9?

369 396 639 693 936 963

Example: A coin is flipped, and a dice is rolled. List all the possible outcomes.

A head on the coin → H 1 H 2 H 3 H 4 H 5 H 6 ← A tail on the coin

A 1 on the dice → T 1 T 2 T 3 T 4 T 5 T 6 ← A 6 on the dice

Sample Space Diagram

A sample space diagram is a way of showing multiple outcomes in one diagram.

Example: Two dice are thrown, and the numbers are multiplied together. The table below shows some of the possible outcomes.

Second Dice (rows) / First Dice (columns)

Second Dice						
6	6	12	18	24	30	36
5	5	10	15	20	25	30
4	4	8	12	16	20	24
3	3	6	9	12	15	18
2	2	4	6	8	10	12
1	1	2	3	4	5	6
	1	2	3	4	5	6

First Dice

First dice x second dice 5 x 6 = 30

First dice x second dice 6 x 3 = 18

a) Complete the table to show **all** the possible outcomes.

b) What is the probability of getting an outcome that is an odd number?

$P(\text{odd}) = \frac{9}{36} = \frac{1}{4}$

Number of outcomes that are odd numbers

Total number of outcomes

c) If the two dice were thrown a total of 60 times, how many times would you expect to get an odd number?

$P(\text{odd}) = 60 \times \frac{9}{36} = 15$ times

Number of times the dice are thrown

Probability of an odd number

Finding Missing Probabilities from a Table

Probabilities add up to 1, to find the missing probabilities add together the probabilities you are given and subtract them from 1.

Example: A biased spinner has 4 colours. The probability of the spinner landing on each colour is given below.

Colour	Red	Blue	Yellow	Green
Number of times	0.1	x	0.4	0.2

a) What is the probability of choosing a blue sweet?

P(Blue) = 0.1 + 0.4 + 0.2 = 0.7 ← Add the probabilities

1 - 0.7 = 0.3 ← Subtract them from 1

b) The spinner is spun 100 times. Calculate an estimate for the number of times the spinner will land on yellow.

P(Yellow) = 100 x 0.4 = 40 times

Number of times the spinner is spun

Probability of a yellow

Mathematics

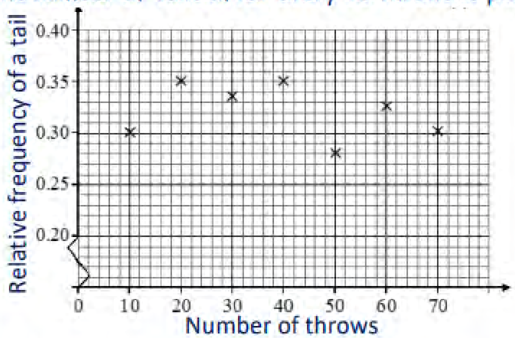
Intermediate

Unit 42

Relative frequency from a graph

Example:

A coin is thrown 70 times. The relative frequency of the number of tails after every 10 throws is plotted.



a) How many tails were obtained in 50 throws?

0.28 x 50 = 14 tails

Relative frequency of 50

Number of throws

b) Use the diagram to estimate the probability of getting a tail.

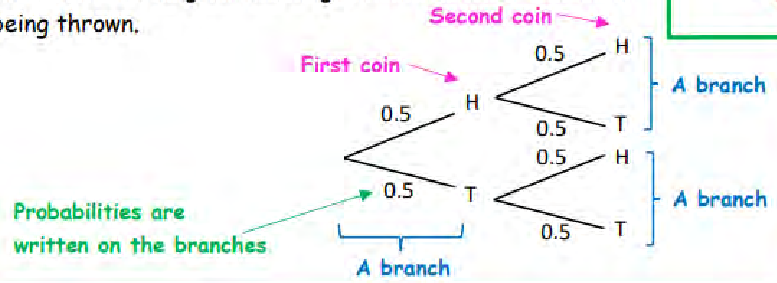
P(T) = 0.3 (the more throws the more accurate the result, 70 throws = 0.3)

Probability Trees

Tree diagrams are a way of showing combinations of two or more events.

Tree diagrams have branches, with each branch adding to 1.

Here is a tree diagram showing the outcomes of two coins being thrown.



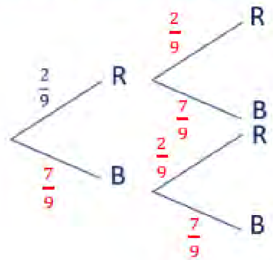
Example:

A bag contains red and blue counters. The probability that a red counter is chosen is $\frac{2}{9}$.

A counter is chosen and replaced; a second counter is chosen.

a) Complete the tree diagram below.

remember each branch adds to 1



b) Calculate the probability that a red counter is chosen followed by a blue counter.

We need to follow the branches, red for the first counter **AND** blue for the second counter.

P(R **and** B) $= \frac{2}{9} \times \frac{7}{9} = \frac{14}{81}$

c) Calculate the probability that two counters of the same colour are chosen.

P(R **and** R) $= \frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$

OR

P(B **and** B) $= \frac{7}{9} \times \frac{7}{9} = \frac{49}{81}$

$\frac{4}{81} + \frac{49}{81} = \frac{53}{81}$